Pleasant Street Primary School and Nursery

Calculations Policy

Facilitating a 'concrete' experience- and using models and images.

Bar Model

191	192	193	194	195	196	197	198	199	200
181	182	183	184	185	186	187	188	189	190
171	172	173	174	175	176	177	178	179	180
161	162	163	164	165	166	167	168	169	170
151	152	153	154	155	156	157	158	159	160
141	142	143	144	145	146	147	148	149	150
131	132	133	134	135	136	137	138	139	140
121	122	123	124	125	126	127	128	129	130
111	112	113	114	115	116	117	118	119	120
101	102	103	104	105	106	107	108	109	110
91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	188	19	20
1	2	3	4	5	6	7	8	9	10

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Background to policy.

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of mathematics. The mental methods for teaching mathematics will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it.

In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at this school children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- Developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- Using standard symbols and conventions;
- Use of jottings to aid a mental strategy;
- Use of pencil and paper procedures;
- Use of a calculator.

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation/problem and then decide the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our long-term aim is for children to be able to select an efficient method of their choice (whether this be mental, written or in upper Key Stage 2 using a calculator) that is appropriate for a given task. They will do this by always asking themselves:

- 'Can I do this in my head?
- 'Can I do this in my head using drawings or jottings?'
- 'Do I need to use a pencil and paper procedure?
- 'Do I need a calculator?

Number and Place Value:
To develop confidence and mental fluency with whole numbers, counting and whole numbers.
Involve working with numerals, words and the four operations, including practical resources.

Year 1 **Expected**	Year 2 **Expected**	Year 3 **Expected**
• **count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number** • **count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s** • **given a number, identify 1 more and 1 less** • identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least • read and write numbers from 1 to 20 in numerals and words	• **count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward** • recognise the place value of each digit in a two-digit number (10s, 1s) • identify, represent and estimate numbers using different representations, including the number line • **compare and order numbers from 0 up to 100; use <, > and = signs** • read and write numbers to at least 100 in numerals and in words • **use place value and number facts to solve problems**	• **count from 0 in multiples of 4, 8, 50 and 100** • **find 10 or 100 more or less than a given number** • **recognise the place value of each digit in a 3-digit number (100s, 10s, 1s)** • compare and order numbers up to 1,000 • identify, represent and estimate numbers using different representations • read and write numbers up to 1,000 in numerals and in words • **solve number problems and practical problems involving these ideas**
In-depth	**In-depth**	**In-depth**
• practise counting (1, 2, 3…), ordering (for example, first, second, third…), and to indicate a quantity (for example, 3 apples, 2 centimetres), including solving simple concrete problems, until they are fluent. • begin to recognise place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100, supported by objects and pictorial representations. • practise counting as reciting numbers and counting as enumerating objects, and counting in 2s, 5s and 10s from different multiples to develop their recognition of patterns in the number system (for example, odd and even numbers), including varied and frequent practice through increasingly complex questions. • recognise and create repeating patterns with objects and with shapes.	• use materials and a range of representations, pupils practise counting, reading, writing and comparing numbers to at least 100 and solving a variety of related problems to develop fluency. • count in multiples of 3 to support their later understanding of a third. • become more confident with numbers up to 100, introduce larger numbers to develop further recognition of patterns within the number system and represent them in different ways, including spatial representations. • partition numbers in different ways (for example, 23 = 20 + 3 and 23 = 10 + 13) to support subtraction. • become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that emphasise the value of each digit in two-digit numbers. • begin to understand 0 as a place holder.	• use multiples of 2, 3, 4, 5, 8, 10, 50 and 100. • use larger numbers to at least 1,000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (for example, 146 = 100 + 40 + 6, 146 = 130 +16). • use a variety of representations, including those related to measure, • pupils continue to count in 1s, 10s and 100s, so that they become fluent in the order and place value of numbers to 1,000.

Addition		
Year 1 **Expected**	Year 2 **Expected**	Year 3 **Expected**
• read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs • **represent and use number bonds and related subtraction facts within 20** • add and subtract one-digit and two-digit numbers to 20, including 0 • Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? − 9	• **solve problems with addition and subtraction:** o **using concrete objects and pictorial representations, including those involving numbers, quantities and measures** o **applying their increasing knowledge of mental and written methods** • **recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100** • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: o a two-digit number and 1s o a two-digit number and 10s o 2 two-digit numbers o adding 3 one-digit numbers • show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems	• **add and subtract numbers mentally, including:** **a three-digit number and 1s** **a three-digit number and 10s** **a three-digit number and 100s** • add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction
In-depth	**In-depth**	**In-depth**
• memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 − 7 = 9; 7 = 16 − 9). • realise the effect of adding or subtracting 0. • establish addition and subtraction as related operations. • combine and increase numbers, counting forwards and backwards. • discuss and solve problems in familiar practical contexts, including using quantities. • Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.	• extend their understanding of the language of addition and subtraction to include sum and difference. • practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3 + 7 = 10; 10 − 7 = 3 and 7 = 10 − 3 to calculate 30 + 70 = 100; 100 − 70 = 30 and 70 = 100 − 30. • check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition. • record addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.	• practise solving varied addition and subtraction questions. • for mental calculations with two-digit numbers, the answers could exceed 100. • use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to 3 digits to become fluent

	Addition	
Stage 1	**Stage 2**	**Stage 3**

+ = signs and missing numbers

$3 + 4 = \square$ $\square = 3 + 4$
$3 + \square = 7$ $7 = \square + 4$
$\square + 4 = 7$ $7 = 3 + \square$
$\square + \nabla = 7$ $7 = \square + \nabla$

3 + 4 is the same as 7 as modelled using Numicon
Use Numicon to further understand the equivalence in a number sentence.
Promoting covering up of operations and numbers.
Using Number lines
(Teacher model number tracks and lines with numbers and with missing numbers)

7 + 4 = 11 Children go up in 1s

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Number and Place Value Extend the knowledge of the number system and place value to include large integers Develop an ability to solve a wide range of problems demanding efficient written methods and mental methods of calculation		
Year 4 **Expected**	**Year 5** **Expected**	**Year 6** **Expected**
• **count in multiples of 6, 7, 9, 25 and 1,000** • find 1,000 more or less than a given number • **count backwards through 0 to include negative numbers** • recognise the place value of each digit in a four-digit number (1,000s, 100s, 10s, and 1s) • **order and compare numbers beyond 1,000** • identify, represent and estimate numbers using different representations • **round any number to the nearest 10, 100 or 1,000** • solve number and practical problems that involve all of the above and with increasingly large positive numbers • read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of 0 and place value • count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten. • recognise and write decimal equivalents of any number of tenths or hundredths	• **read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit** • count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000 • **interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through 0** • round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000 • solve number problems and practical problems that involve all of the above • read Roman numerals to 1,000 (M) and recognise years written in Roman numerals • read and write decimal numbers as fractions [for example, 0.71 = 71/100] • recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents • round decimals with two decimal places to the nearest whole number and to one decimal place • read, write, order and compare numbers with up to three decimal places	• read, write, order and compare numbers up to 10,000,000 and determine the value of each digit • **round any whole number to a required degree of accuracy** • **use negative numbers in context, and calculate intervals across 0** • solve number and practical problems that involve all of the above
In-depth	**In-depth**	**In-depth**
• use a variety of representations, including measures. • become fluent in the order and place value of numbers beyond 1,000, including counting in 10s and 100s. • maintaining fluency in other multiples through varied and frequent practice. • begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far. • connect estimation and rounding numbers to the use of measuring instruments. • Roman numerals should be put in their historical context so pupils understand that there have been different ways to write whole numbers and that the important concepts of 0 and place value were introduced over a period of time • know that decimals and fractions are different ways	• identify the place value in large whole numbers. • continue to use number in context, including measurement. • extend and apply their understanding of the number system to the decimal numbers and fractions that they have met so far. • recognise and describe linear number • sequences (for example, 3, 31/2, 4, 41/2 …), including those involving fractions and decimals, and find the term-to-term rule in words (for example, add). • extend counting from year 4, using decimals and fractions including bridging zero, for example on a number line. • read and write decimal fractions and related tenths, hundredths and thousandths accurately and are confident in checking the reasonableness of their answers to problems.	• use the whole number system, including saying, reading and writing numbers accurately.

- of expressing numbers and proportions.
- understand of the number system and decimal place value is extended at this stage to tenths and then hundredths.
- includes relating the decimal notation to division of whole number by 10 and later 100.
- practise counting using simple fractions and decimals, both forwards and backwards.
- learn decimal notation and the language associated with it, including in the context of measurements.
- make comparisons and order decimal amounts and quantities that are expressed to the same number of decimal places.
- should be able to represent numbers with one or two decimal places in several ways, such as on number lines.

Addition		
Year 4 **Expected**	Year 5 **Expected**	Year 6 **Expected**
- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate - estimate and use inverse operations to check answers to a calculation **solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why** - round decimals with one decimal place to the nearest whole number - compare numbers with the same number of decimal places up to two decimal places - solve simple measure and money problems involving fractions and decimals to two decimal places.	**add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)** **add and subtract numbers mentally with increasingly large numbers** - use rounding to check answers to calculations and determine, in the context of a problem, accuracy - solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why - solve problems involving number up to three decimal places - recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal - solve problems which require knowing percentage and decimal equivalents of ½, ¼, 1/5,2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25.	- perform mental calculations, including with mixed operations and large numbers - identify common factors, common multiples and prime numbers - use their knowledge of the order of operations to carry out calculations involving the 4 operations **solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why** - solve problems involving addition, subtraction, multiplication and division **use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy** - solve problems which require answers to be rounded to specified degrees of accuracy
In-depth	**In-depth**	**In-depth**
continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency	- practise using the formal written methods of - columnar addition with increasingly large numbers to aid fluency. - practise mental calculations with increasingly large numbers, e.g., 12,462 – 2,300 = 10,162). - introduce the language of algebra as a means of solving a variety of problems - mentally add and subtract tenths, and one-digit whole numbers and tenths. - practise adding and subtracting decimals, including a mix of whole numbers and decimals, with different numbers of decimal places, and complements of 1 (for example, 0.83 + 0.17 = 1). - Pupils should go beyond the measurement and money models of decimals to solving puzzle	- practise addition for larger numbers, using the formal written methods of columnar addition - undertake mental calculations with increasingly large numbers and more complex calculations. - round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50, etc, but not to a specified number of significant figures. - explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9. - Understand common factors can be related to finding equivalent fractions - develop the language of algebra as a means of solving a variety of problems.

Addition		
Stage 4	**Stage 5**	**Stage 6**

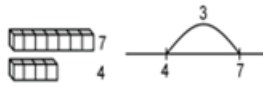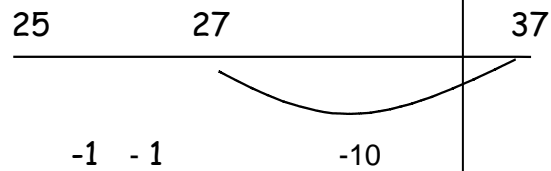| Subtraction		
Year 1 **Expected**	**Year 2** **Expected**	**Year 3** **Expected**
• **<u>count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number</u>** • **<u>given a number, identify 1 more and 1 less</u>** identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least	• **<u>solve problems with addition and subtraction:</u>** o **<u>using concrete objects and pictorial representations, including those involving numbers, quantities and measures</u>** o **<u>applying their increasing knowledge of mental and written methods</u>** • **<u>recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</u>** • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: o a two-digit number and 1s o a two-digit number and 10s o 2 two-digit numbers o adding 3 one-digit numbers • show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems	• **<u>add and subtract numbers mentally, including:</u>** **<u>a three-digit number and 1s</u>** **<u>three-digit number and 10s</u>** **<u>a three-digit number and 100s</u>** • add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction
In-depth	**In-depth**	**In-depth**
• memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 − 7 = 9; 7 = 16 − 9). • realise the effect of adding or subtracting 0. This establishes addition and subtraction as related operations. • combine and increase numbers, counting forwards and backwards. • discuss and solve problems in familiar practical contexts, including using quantities. problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.	• extend their understanding of the language of addition and subtraction to include sum and difference. • practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3 + 7 =10; 10 − 7 = 3 and 7 = 10 − 3 to calculate 30 + 70 = 100; 100 − 70 = 30 and 70 = 100 − 30. • check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition. • record addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers	• practise solving varied addition and subtraction questions. • mental calculations with two-digit numbers, the answers could exceed 100. • use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to 3 digits to become fluent • develop the language of algebra as a means of solving a variety of problems

Subtraction

Stage 1	Stage 2	Stage 3

Stage 1

numbers
$7 - 3 = \square$ $\square = 7 - 3$
$7 - \square = 4$ $4 = \square - 3$
$\square - 3 = 4$ $4 = 7 - \square$
$\square - \nabla = 4$ $4 = \square - \nabla$

Stage 2

25   27   37

-1   - 1   -10

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

$37 - 23 = ?$

$37 - 20 =$
17

$17 - 3 = 14$

Bar   Model

37

| 23 | | ? |
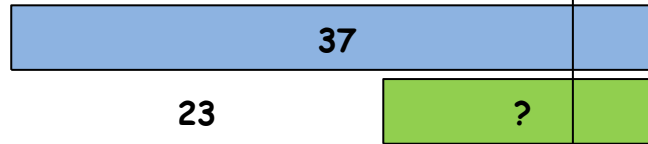
Consolidate counting on as a
strategy when the numbers
are close together.

Stage 3

Counting back on a number line.

35   37   67

- 2   -30
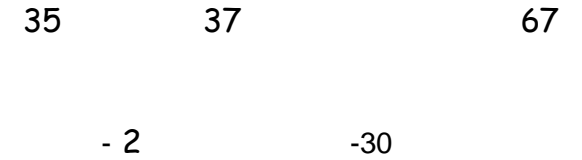
Expanded method of subtraction.

Expanded method with carrying.

Bar Method

Subtraction		
Year 4 **Expected**	**Year 5** **Expected**	**Year 6** **Expected**
• add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • **solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why** • solve simple measure and money problems involving fractions and decimals to two decimal places.	• **add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)** • **add and subtract numbers mentally with increasingly large numbers** • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	• use their knowledge of the order of operations to carry out calculations involving the 4 operations • **solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why** • solve problems involving addition, subtraction, multiplication and division • **use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy** • perform mental calculations, including with mixed operations and large numbers • solve problems which require answers to be rounded to specified degrees of accuracy
In-depth	**In-depth**	**In-depth**
• continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency	• practise using the formal written methods of columnar subtraction with increasingly large numbers to aid fluency. • practise mental calculations with increasingly large numbers to aid fluency (for example, 12,462 – 2,300 = 10,162). • introduce the language of algebra as a means of solving a variety of problems • mentally add and subtract tenths, and one-digit whole numbers and tenths.	• practise subtraction for larger numbers, using the formal written methods of columnar subtraction, • undertake mental calculations with increasingly large numbers and more complex calculations. • develop the language of algebra as a means of solving a variety of problems

Subtraction		
Stage 4	**Stage 5**	**Stage 6**
Formal Method for Subtraction. Place Value titles to be shown in all calculations. Ones to be used. Formal Method with exchanging. Formal Method with 0 as a place holder. Subtraction must start from the right. Bar Method	Formal Method for Subtraction using more than 4 digits. Formal Method for Subtraction involving decimals up to two places. Bar Method	Can you use five of the digits 1 to 9 to make this number sentence true? $.\quad - \quad.\quad = 23.8$ For other examples see Mastery documentation NCETM and progression maps for reasoning. (Upper KS2.)

Multiplication		
Year 1 **Expected**	Year 2 **Expected**	Year 3 **Expected**
• solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	• **recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers** • calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs • show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot • **solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts**	• **recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.** • **write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods** • solve problems, including missing number problems, involving multiplication and division, including positive interger scaling problems and correspondence problems in which n objects are connected to m objects
In-depth	**In-depth**	**In-depth**

• through grouping and sharing small quantities, pupils begin to understand:
  • multiplication and division
  • doubling numbers and quantities
  • finding simple fractions of objects, numbers and quantities
• make connections between arrays, number patterns, and counting in twos, fives and tens.
	• use a variety of language to describe multiplication and division
	• introduce the multiplication tables.
	• practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other.
	• connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face.
	• begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.
	• Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition.
	• begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40).
	• use inverse relations to develop reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).
		• develop efficient mental methods, for example, using commutativity and associativity (for example, 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (eg, using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (eg, 30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
		• develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.
		• solve simple problems in contexts, deciding which of the four operations to use and why. Include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (eg, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Multiplication		
Stage 1	**Stage 2**	**Stage 3**

Multiplication		
Year 4 **Expected**	**Year 5** **Expected**	**Year 6** **Expected**
• **recall multiplication and division facts for multiplication tables up to 12 × 12** • use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects	• **identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers** • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers • establish whether a number up to 100 is prime and recall prime numbers up to 19 • multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally, drawing upon known facts • multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 • recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) • **solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes** • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • **solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates**	• **multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication** • identify common factors, common multiples and prime numbers • perform mental calculations, including with mixed operations and large numbers • use their knowledge of the order of operations to carry out calculations involving the 4 operations • **solve problems involving addition, subtraction, multiplication and division** • **use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy** • identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places • multiply one-digit numbers with up to two decimal places by whole numbers • solve problems which require answers to be rounded to specified degrees of accuracy • recall and use equivalences between simple fractions, decimals and percentages, including in different contexts. • solve problems which require answers to be rounded to specified degrees of accuracy
In-depth	**In-depth**	**In-depth**
• continue to practise recalling and using multiplication tables and related division facts to aid fluency. • practise mental methods and extend this to	• practise and extend their use of the formal written methods of short multiplication and short division. • apply all the multiplication tables and related division	• practise multiplication for larger numbers, using the formal written methods of short and long multiplication. • undertake mental calculations with increasingly

3-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).

- practise to become fluent in the formal written method of short multiplication and short division with exact answers

- write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)).

- combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.

- solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or 3 cakes shared equally between 10 children

facts frequently, commit them to memory and use them confidently to make larger calculations.

- use and understand the terms factor,
- multiple and prime, square and cube numbers.
- interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, 98 ÷ 4 = 98/4 = 24 r 2 = 241/2 = 24.5 ≈ 25).
- use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1,000 in converting between units such as kilometres and metres.
- introduce the language of algebra as a means of solving a variety of problems distributivity can be expressed as a(b + c) = ab + ac.
- understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9² x 10).
- use and explain the equals sign to indicate equivalence, including in missing number problems (for example 13 + 24 = 12 + 25; 33 = 5 x ?).

large numbers and more complex calculations.

- Continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.
- explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.
- understand common factors can be related to finding equivalent fractions
- develop the connection made between multiplication and division with fractions, decimals, percentages and ratio
- develop the language of algebra as a means of solving a variety of problems
- multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers.
- multiply decimals by whole numbers, starting with the simplest cases, such as 0.4 × 2 = 0.8, and in practical contexts, such as measures and money.

Multiplication		
Stage 4	**Stage 5**	**Stage 6**

Division		
Year 1 **Expected**	**Year 2** **Expected**	**Year 3** **Expected**
• solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	• **recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers** • calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs • show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot • **solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts**	• **recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.** • **write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods** • solve problems, including missing number problems, involving multiplication and division, including positive interger scaling problems and correspondence problems in which n objects are connected to m objects
In-depth	**In-depth**	**In-depth**

• through grouping and sharing small quantities, pupils begin to understand:
  • multiplication and division
  • doubling numbers and quantities
  • finding simple fractions of objects, numbers and quantities
• make connections between arrays, number patterns, and counting in twos, fives and tens.
	• use a variety of language to describe multiplication and division.
	• begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.
	• work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition.
	• begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40).
	• use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).
		• develop efficient mental methods, for example, using commutativity and associativity (for example, 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (eg, using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (eg, 30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
		• develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.
		• solve simple problems in contexts, deciding which of the four operations to use and why. Include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (eg, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Division		
Stage 1	**Stage 2**	**Stage 3**

Division		
Year 4 **Expected**	**Year 5** **Expected**	**Year 6** **Expected**
recall multiplication and division facts for multiplication tables up to 12 x 12 - use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers - recognize and use factor pairs and commutativity in mental calculations - find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths	**identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers** - multiply and divide numbers mentally, drawing upon known facts - divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context - multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 - recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) **solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes** - solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign **solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rate**	- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context **divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context** - use their knowledge of the order of operations to carry out calculations involving the 4 operations **solve problems involving addition, subtraction, multiplication and division** **use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy** - associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8] - identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places - use written division methods in cases where the answer has up to two decimal places - solve problems which require answers to be rounded to specified degrees of accuracy
In-depth	**In-depth**	**In-depth**
- continue to practise recalling and using multiplication tables and related division facts to aid fluency. - practise mental methods and extend this	- practise and extend their use of the formal written methods of short multiplication and short division. - apply all the multiplication tables and	- practise division for larger numbers, using the formal written methods of short and long division - explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.

- to 3-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).

- practise to become fluent in the formal written method of short multiplication and short division with exact answers
- solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or 3 cakes shared equally between 10 children

- related division facts frequently, commit them to memory and use them confidently to make larger calculations.

- use and understand the terms factor, multiple and prime, square and cube numbers.
- interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, 98 ÷ 4 = 98/4 = 24 r 2 = 24½ = 24.5 ≈ 25).
- use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1,000 in converting between units such as kilometres and metres.

- Understand common factors can be related to finding equivalent fractions
- develop the connection made between

  multiplication and division with fractions, decimals, percentages and ratio
- explore and make conjectures about converting a simple fraction to a decimal fraction (for example, 3 ÷ 8 = 0.375).
- For simple fractions with recurring decimal equivalents, pupils learn about rounding the decimal to three decimal places, or other appropriate approximations depending on the context.
- introduce to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money.

- recognise division calculations as the inverse of multiplication.
- develop the skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations, including rounding answers to a specified degree of accuracy and checking the reasonableness of their answers.

Division		
Stage 4	**Stage 5**	**Stage 6**